Lesson 28: Antiderivatives and Indefinite Integrals

Def A differential equation is an equation including variables (x, y, etc.) and derivatives ($\frac{dy}{dx}$, etc.)

Ex 1 $\frac{dy}{dx} = \sec^2 x$

$y = \int \sec^2 x \, dx$

$\boxed{y = \tan x + C}$

Ex 2 $y' = \frac{x+1}{x^2}$

$y = \int \frac{x+1}{x^2} dx$

$= \int \frac{1}{x} + x^{-2} dx$

$\boxed{y = \ln|x| - x^{-1} + C}$

Def An initial value problem is a differential equation with an initial condition.

Ex 3 $\underbrace{f'(t) = \sec t \tan t}_{\text{differential equation}}$ and $\underbrace{f(\pi) = 1}_{\text{initial condition}}$.

$f(t) = \int \sec t \tan t \, dt$

$f(t) = \sec t + C$

At $t = \pi$:

$f(\pi) = \sec \pi + C \overset{set}{=} 1$

$C = 1 - \sec \pi$

$= 1 - (-1) = 2$

$\boxed{f(t) = \sec t + 2}$

Ex 4 $g'(x) = \frac{1}{x}$ and $g(e^2) = 3$

$g(x) = \int \frac{1}{x} dx$

$= \ln|x| + C$

$g(e^2) = \ln|e^2| + C \overset{set}{=} 3$

$2 + C = 3$

$C = 1$

$\boxed{g(x) = \ln|x| + 1}$

$\ln e^2 = 2(\cancel{\ln e})^{1} = 2$

Ex 5 $\underbrace{g''(x)=x}_{\text{diff. eq}}$, $\underbrace{g'(4)=10,\ g(-6)=3}_{\text{initial conditions}}$

$g'(x) = \int x\,dx$

$= \frac{1}{2}x^2 + C$

$g'(4) = \frac{1}{2}(4)^2 + C \overset{\text{set}}{=} 10$

$8 + C = 10$

$C = 2$

$g'(x) = \frac{1}{2}x^2 + 2$

$g(x) = \int \frac{1}{2}x^2 + 2\,dx$

$= \frac{1}{2}\left(\frac{1}{3}x^3\right) + 2x + C$

$= \frac{1}{6}x^3 + 2x + C$

$g(-6) = \frac{1}{6}(-6)^3 + 2(-6) + C \overset{\text{set}}{=} 3$

$C = 51$

$\boxed{g(x) = \frac{1}{6}x^3 + 2x + 51}$

Ex 6 The rate of change $\frac{dy}{dx}$ is proportional to the cube of x. If $y(0)=10$ and $y(1)=11$, find $y(2)$.

$\frac{dy}{dx} = kx^3$ constant

$\frac{dy}{dx} = kx^3$

$y = \int kx^3\,dx$

$= k\int x^3\,dx$

$= k\left(\frac{1}{4}x^4\right) + C$

$y = \frac{1}{4}kx^4 + C$

$y(0) = \cancel{\frac{1}{4}k(0)^4} + C \overset{\text{set}}{=} 10$

$C = 10$

$y = \frac{1}{4}kx^4 + 10$

$y(1) = \frac{1}{4}k(1)^4 + 10 \overset{\text{set}}{=} 11$

$\frac{1}{4}k = 1$

$k = 4$

$y = \frac{1}{4}(4)x^4 + 10$

$\boxed{y = x^4 + 10}$

Ex 7 A space shuttle's booster detaches when the shuttle reaches an altitude of 4500m $\rightarrow s(0)$, at which point its velocity is 1380 m/s $\rightarrow v(0)$. The acceleration due to gravity is $-9.8\ m/s^2 \rightarrow a(t)$.

Find $p(t)$, the height of the booster t seconds after it detaches.

$\underbrace{p''(t) = a(t) = -9.8}_{\text{diff. eq.}}$, $p(0) = 4500$, $p'(0) = v(0) = 1380$

$$p'(t) = \int -9.8\, dt$$

$$p'(t) = -9.8t + C$$

$$p'(0) = -9.8(0) + C = 1380$$

$$C = 1380$$

$$p'(t) = -9.8t + 1380$$

$$p(t) = \int -9.8t + 1380\, dt$$

Intro to Lesson 29:

$$1 + 2 + 3 + 4 + 5 + 6 = \sum_{i=1}^{6} i$$

$$\sum_{i=2}^{4} \frac{i}{3} = \frac{2}{3} + \frac{3}{3} + \frac{4}{3}$$